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Modulational instability of lower hybrid waves at the magnetopause

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Abstract. The role of lower hybrid waves at the magnetopause is reexamined. It is found that for the maximum observed wave power, the lower hybrid waves are unstable to a modulational instability on the magnetosheath side of the magnetopause. The modulational instability leads to localized field structures oriented predominantly along the magnetic field. Such patchy lower hybrid turbulence has been observed by some spacecraft. As a result of the large T_i/T_e ratio, the waves saturate by ion heating; as a result, unlike other settings (e.g. comets, critical ionization phenomena), energetic electrons are not expected. The stochastic electron transport in the presence of such turbulence is analyzed and results in strongly anisotropic electron diffusion, with the dominant direction across the magnetic field. The diffusion rate exceeds significantly that expected from quasi-linear considerations and, for magnetopause parameters, also exceeds the rate discussed by Sonnerup (1980).

1. Introduction

The magnetopause is one of the key boundary layers responsible for determining the overall magnetospheric structure. It is the interface between the solar and terrestrial magnetic fields and is thus responsible for regulating the flow of mass and energy from the solar wind to the magnetosphere. Two distinct types of magnetopause structures are possible, depending upon whether mass can access the magnetosphere directly or not. If material can flow directly through the magnetopause, there is a direct connection between the solar and terrestrial magnetic fields, and the magnetopause is said to be "open". On the other hand, if there is no topological connection, the magnetopause is said to be "closed", and any material accessing the magnetosphere must somehow "diffuse" across the magnetic field. The present paper is only concerned with the latter class of magnetopause behavior; the open magnetopause will not be further discussed.

When the magnetopause is closed, it resembles a classic tangential discontinuity (TD). A TD is one of the four fundamental classes of discontinuity that can exist in a magnetohydrodynamic description of a plasma (the others being shocks, rotational and contact discontinuities). It is characterized by the absence of normal plasma flow and magnetic field components. The only condition that must be satisfied is that the sum of the plasma and magnetic pressure be constant. Such structures have been investigated theoretically [*Cargill and Eastman*, 1991; *Thomas and Winske*, 1993] and it was found that finite Larmor radius effects play a key role in determining their equilibrium structure.

Observations at the Earth's magnetopause reveal the existence of intense lower hybrid (LH) turbulence (see reviews by *Labelle and Treumann*, [1988] and *Treumann et al.* [1992]). For example, ISEE measurements carried out by *Gurnett et al.* [1979] show considerable wave intensity at frequencies close to the LH frequency (30 - 50 Hz) during magnetopause crossings [see *Gurnett et al.*, 1979; Figure 15]. In some cases, Gurnett et al. note a break in the slope of the spectrum at around 50 Hz, which they attribute to its proximity to the lower hybrid frequency. The free energy responsible for the excitation of these waves has a number of possible sources, but the lower hybrid drift instability (LHDI) driven by the cross-field diamagnetic current has been the most studied [e.g., *Gary and Eastman*, 1979; *Sotnikov et al.*, 1980; *Gary and Sgro*, 1990; *Winske et al.*, 1991; *Treumann et al.*, 1991, 1992]. Hybrid simulations of TDs have also

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shown that the cross-field drifts associated with these currents can reach significant fractions of the ion thermal speed [*Cargill and Eastman*, 1991].

An issue of particular interest is whether the LHDI can give rise to adequate anomalous transport of matter to populate the low-latitude boundary layer. *Sonnerup* [1980] provided an estimate for the required diffusion rate based on global magnetospheric considerations, and quotes a diffusion coefficient of 10^9 m^2/s. *Sotnikov et al.* [1980], *Treumann et al.* [1991], and *Winske et al.* [1991] argued that the LHDI could provide adequate diffusion. However, particle simulations of *Winske et al.* [1991] showed that a large-scale transport of matter did not occur and that magnetosheath (magnetospheric) ions tended to stay in the magnetosheath (magnetosphere).

Simulations and theory of the nonlinear stages of the LHDI have generally concentrated on evolution by either current relaxation and (kinetic) interaction with the electrons, or ion trapping [see *Davidson*, 1978; *Winske and Liewer*, 1978; *Gary and Eastman*, 1979; *Gary*, 1980; *Sotnikov et al.*, 1980; *Brackbill et al.*, 1984; *Sgro and Gary*, 1990 and *Winske et al.*, 1991]. However, there are also indications that the LHDI can evolve by the collapse of LH wave packets due to a modulational instability [*Musher and Sturman*, 1975; *Sotnikov et al.*, 1978; *Shapiro et al.*, 1993b]. This collapse occurs when the amplitude of the LH turbulence exceeds a critical value and leads to ion and electron heating as well as the possibility of energetic electron tails. (It should be noted that a number of numerical simulations of the LHDI would not expect to see the modulational instability due to their choice of a simulation geometry that automatically precludes it [*Sgro and Gary*, 1990; *Winske et al.*, 1991].)

It is the purpose of this paper to investigate whether the observed LH turbulence at the magnetopause is susceptible to the modulational instability, and to examine the consequences of the modulational instability on the magnetopause structure and transport. Section 2 presents a very brief summary of the linear analysis of the LHDI. Section 3 discusses the conditions for the onset of the modulational instability, and Section 4 examines the resultant electron diffusion rate across the magnetopause.

2. Existing Linear Stability Theory and Saturation Mechanisms

The linear theory of lower hybrid-type instabilities is well known [*Davidson et al.*, 1977; *Huba et al.*, 1978; *Hsia et al.*, 1979], so only a brief summary is required here. In this paper we concentrate on waves generated by cross-field currents due to ion or electron diamagnetic drifts and ignore the beam generated modes that are usually discussed in terms of the modified two-stream instability [*McBride and Ott*, 1972].

Using hybrid numerical simulations, *Cargill and Eastman* [1991] demonstrated that the cross-field drift at a TD for typical magnetopause parameters could be a significant fraction of, and in some cases could exceed, the ion thermal speed (see Figure 2 of that paper). For $V_D \ll V_i$ and a unidirectional magnetic field, the frequency, growth rate and least stable wavenumber are given by [*Huba et al.*, 1978]

$$\omega_0 \approx \frac{1}{\sqrt{2}}\left(\frac{V_D}{V_i}\right)\omega_{LH} \quad , \quad \gamma_0 \approx \omega_0 \frac{\sqrt{\pi}}{4}\left(\frac{\kappa \rho_{Li}}{2}\right) \quad , \tag{1}$$

$$k_0 \approx \sqrt{2}\frac{\omega_{LH}}{V_i} \quad , \tag{2}$$

where the subscript 0 corresponds to the least stable mode, $\omega_{LH} = (\Omega_e \Omega_i)^{1/2}$ is the lower hybrid frequency in the regime where $\omega_e \gg \Omega_e$ and ρ_{Li} is the ion Larmor radius. Here $\omega_j(\Omega_j)$ is the plasma (cyclotron) frequency of the jth species. We have expressed the growth rate in terms of $\kappa = |dn_0/dx|/n_0$ to emphasize the role the density gradient plays in driving the LHDI. Although strictly only valid for $V_D \ll V_i$, *Huba et al.* [1978] note that (1) and (2) are accurate up to $V_D \simeq V_i$. For $V_D \gg V_i$, the instability is fluidlike, and $\omega \sim \gamma \sim \omega_{LH}$.

We note here that the presence of a B_y (east-west) magnetic field has a stabilizing influence on the properties of the LHDI [*Huba and Ganguli*, 1983; *Huba et al.*, 1983]. The stabilization mechanism in such a geometry is due to the presence of a parallel wave vector, $k_\parallel \sim k_y(x/L_s)$ (where L_s is the typical shear scale length). This $k_\parallel$ stabilizes the LHDI due to electron Landau damping. If $\omega \sim k_\parallel V_e$, then the LH waves are localized into field-aligned filaments with a cross-field dimension of approximately $L_s(V_D/V_e)$, which is comparable or larger than the cross-field growth length of the instability since the group velocity of the LH waves is almost field-aligned. The amount of the stabilization depends on the precise details of the chosen equilibria, but in general, the density gradient must be shorter than the gradient in the sheared field for instability to persist. This appears frequently to be the case (T. Eastman, private communication, 1991).

A number of theories have been proposed for the nonlinear evolution of the LHDI. In the regime $V_D < V_i$, many workers have found that

$$E_s = \frac{E^2}{8\pi nT} \approx A\left(\frac{\Omega_i^2}{\omega_i^2}\right)\left(\frac{V_D^2}{V_i^2}\right) ,$$

where A is a constant, typically smaller than unity. Examples of this are a current relaxation model of *Davidson* [1978], an orbit modification model of *Gary and Sanderson* [1979] and a mechanism that relies on mode coupling to short-wavelength damped modes [*Drake et al.*, 1984]. In the high V_D regime, *Winske and Liewer* [1978] argue that the instability can saturate by ion trapping, and *Huba et al.*, [1978] give

$$E_s \approx 2 \times 10^{-2}\left(\frac{k_0^2 V_i^2}{\omega_i^2}\right)\left(1 + \frac{2\omega_0^2}{k_0^2 V_i^2}\right)^2 .$$

3. Nonlinear Evolution: Modulational Instability

It has been well known for many years that when the amplitude of microturbulence exceeds a given value, the waves are unstable to a modulational instability (MI) that changes the homogeneous structure of the turbulence to an inhomogeneous one. This cannot be described by the traditional methods of weak turbulence theory. A strong turbulence theory has been developed for Langmuir turbulence [*Zhakarov*, 1984] as well as for lower hybrid turbulence [*Sotnikov et al.*, 1978; *Shapiro et al.*, 1993b]. In the latter case the MI is described by the following equations:

$$-\frac{2i}{\omega_{LH}}\frac{\partial}{\partial t}\nabla^2\phi - R^2\nabla^4\phi + \frac{m_i}{m_e}\frac{\partial^2\phi}{\partial z^2} = - i\frac{m_i}{m_e}\frac{\omega_{LH}}{\Omega_e n_0}[\nabla\phi\times\nabla n]_{\|} \ , \quad (3)$$

$$\frac{\partial^2\delta n}{\partial t^2} - \frac{(T_e+T_i)}{m_i}\nabla^2\delta n = \frac{i}{4\pi m_i}\frac{\omega_e^2}{\Omega_e\omega_{LH}}\nabla^2[\nabla\phi^*\times\nabla\phi]_{\|} \ , \quad (4)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\phi(\mathbf{r},t)$ is the complex amplitude of the lower hybrid wave potential: $\phi_{LH} = 1/2\phi(\mathbf{r},t)e^{-i\omega_{LH}t} + c.c$, δn is the low-frequency quasi-neutral density perturbation and

$$R^2 = \left(\frac{3}{4} + 3\frac{T_i}{T_e}\right)r_{Le}^2 \ ,$$

[*Sotnikov et al.*, 1978; *Shapiro et al.*, 1993b].

The system (3) and (4) describes the nonlinear coupling of the lower hybrid waves with the slow quasi-neutral density perturbations in the fluid approximation. Kinetic effects leading to wave absorption and electromagnetic corrections to wave dispersion are ignored. Near ω_{LH}, these corrections are of order ω_e^2/k^2c^2 [see *Shapiro et al.*, 1993b]. Comparing them to the dispersive term in (3), we can conclude that electromagnetic corrections are unimportant for short wavelengths that satisfy

$$k^4R^2 > \frac{\omega_e^2}{c^2(1+\beta_e)} \ . \quad (5)$$

Using (2), this can be rewritten as

$$\beta_i < 4\left(\frac{T_e}{T_i}\right)\left[\frac{3}{4} + 3\left(\frac{T_i}{T_e}\right)\right](1+\beta_e) \ ,$$

which is readily satisfied in most cases. Hence electromagnetic (and so finite β_e) effects can be neglected.

A linear analysis of (3) and (4) presented by *Sotnikov et al.* [1978] and *Shapiro et al.* [1993b] gives the condition for instability of the lower hybrid waves as

$$E_s > \frac{1}{2}\frac{\Omega_i^2}{\omega_i^2}k_0^2R^2 \ , \quad (6)$$

where k_0 is the wave number of the lower hybrid wave, defined by (2), and the expression for R can be used to show that

$$k_0^2R^2 = 2\left(\frac{3}{4} + 3\frac{T_i}{T_e}\right)\frac{T_e}{T_i} \quad (7)$$

The same condition arises from the requirement that the ponderomotive force in (3) exceed the dispersion term [*Papadopoulos*, 1992], with the density fluctuations due to the electric field being given by (4). Note that the threshold for the MI exceeds the saturation levels of other LHDI models.

The evolution of the MI differs significantly from conventional homogeneous turbulence theory. Specifically, strong clumping of the plasma and electric field occur over cross-field scales of several wavelengths (a few electron gyroradii). There is no phase coherence between clumps, and so filamentation of the wave electric field into such clumps leads to stabilization of the LHDI. Equation (6) gives the maximum averaged electric field density that can be obtained, although locally the electric field can be much larger.

Estimates for the magnitude of E^2 at the magnetopause can be obtained from spacecraft data. *Gurnett et al.* [1979] discussed lower hybrid waves at the magnetopause and estimated an average electric field strength of order $E^2 \approx 10^{-6}$ V^{2}m^{-2}, with the peak intensity being 1 to 1.5 orders of magnitude larger. They showed a number of cases where the intensity was as high as 5×10^{-5} V^{2}m^{-2} at the 10-Hz level. It should be noted that these intensity levels are obtained simply by multiplying a power spectral density by an approximate frequency. A problem with this approach is that the wave spectrum changes rather smoothly around the lower hybrid frequency, indicating the presence of a broadband spectrum. The intensity levels are thus likely to be very approximate estimates: however, the purpose of this paper is not to carry out detailed comparison with data, but to demonstrate the feasability of the MI instability at the magnetopause. More detailed comparisons will clearly have to pay greater attention to the precise form of the observed spectrum.

More recently, *Tsurutani et al.* [1989] analysed a number of magnetopause crossings. They do not appear to find quite the intensity level of *Gurnett et al.* [1979], but they noted that the emission around the lower hybrid frequency tended to be bursty [*Tsurutani et al.*, 1989, Figure 2], with a sequence of four bursts occurring over a spell of approximately 2 min at the magnetopause crossing. These wave enhancements appear to be present at most magnetopause crossings (85% according to Tsurutani et al.), and the only positive correlation is that the emission is more intense for southward interplanetary magnetic field (IMF). This would seem reasonable since for a given terrestrial field strength, a southward IMF will generate a stronger cross-field current, resulting in a larger source of free energy.

Labelle and Treumann [1988] (see also *Treumann et al.*, [1992]) quote values of E^2 somewhat higher than those cited above. In particular, they quote maximum values of E^2 as large as 10^{-3} V^{2}m^{-2}, far larger than discussed elsewhere. The reason for the discrepency is unclear, although it may be related to the frequencies they considered.

Using (2) for k_0, the stability condition (6) becomes

$$E^2 > 2\frac{B^2}{c^2}\left(\frac{T_i + T_e}{m_i}\right) , \quad (8)$$

In practical units, (8) can be rewritten as

$$E^2 > 1.66 \times 10^{-10} B^2(\gamma)(T_e + T_i)(\text{eV}) \ \text{V}^2\text{m}^{-2} . \quad (9)$$

For typical numbers on the magnetosheath side [*Papamastorakis et al.*, 1984], $B \approx 15\gamma$ (= 1.5×10^{-4}G), $T_i \approx 0.5$ keV, the threshold for MI corresponds to $E^2 > 1.6 \times 10^{-5}$. As a result, the observed lower hybrid turbulence will be modulationally unstable. On the magnetospheric side, where both temperature and magnetic field strengths are higher, the MI is more likely to be supressed.

Having established that the MI can occur, we discuss some of its consequences for lower hybrid waves at the magnetopause. *Sotnikov et al.* [1978] and *Shapiro et al.* [1993a, b] derived self-similar solutions for the wave collapse and showed that the parallel and perpendicular wave numbers scaled as

$$k_\perp^2 R^2 \simeq 2E_s \frac{\omega_i^2}{\Omega_i^2} \quad (10)$$

$$k_\parallel R \simeq \left(\frac{m_e}{m_i}\right)^{1/2} k_\perp^2 R^2 , \quad (11)$$

with $k_\parallel(t) \sim 1/(t - t_0)$. As a result, a wave that has $k = k_0$ initially, collapses into a much smaller scale structure. Equations (6) and (9) established the required values of E_s for the existing lower hybrid waves to be modulationally unstable. Clearly, the observed values of the averaged E^2 correspond approximately to the saturated state of the MI.

The ratio $k_\perp/k_\parallel$ (which can be written as length scales parallel and perpendicular to the field) is

$$\frac{k_\parallel^2}{k_\perp^2} = \frac{L_\perp^2}{L_\parallel^2} = 2\frac{\omega_e^2}{\Omega_e^2} E_s \quad (12)$$

so that for any reasonable choice of parameters, $L_\parallel \gg L_\perp$ and the density cavitons are elongated along the magnetic field. The resulting situation is sketched in Figure 1.

Let us evaluate these expressions for typical parameters in the collapse region. We choose $n = 10$ cm^{-3}, $B = 15\gamma$, $T_i = 0.5$ keV, and $T_e = 30$ eV, corresponding to the magnetosheath side of the magnetopause. Then, for $E^2 = 5 \times 10^{-5}$ V^{2}m^{-2}, we find $E_s = 3 \times 10^{-7}$. Noting that $R \approx 2r_{Le} = 1.6$ km and that $\omega_e/\Omega_e \approx 70$, we find $L_\parallel/L_\perp \approx 18$, $L_\perp \approx 4.3$ km and $L_\parallel \approx 77$ km. If the observed values of E_s do not correspond to the maximum electric field intensities due to, for example, lack of temporal resolution [e.g., *Treumann et al.*, 1992], then

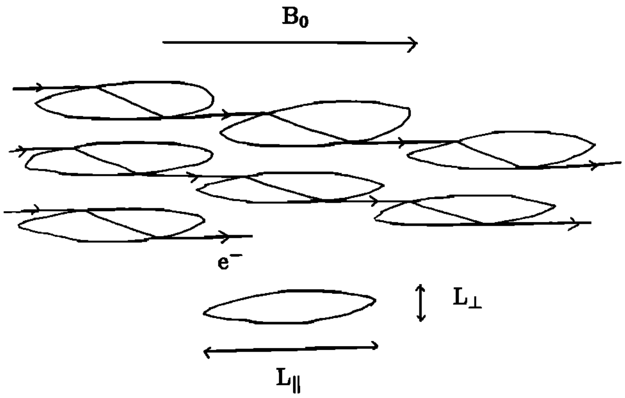

Figure 1. A sketch of the turbulence resulting from the modulational instability of lower hybrid waves. Clumps of turbulence with dimension $L_\perp(l_\parallel)$ across (along) the magnetic field form, with $L_\parallel \gg L_\perp$. The magnetosheath (magnetosphere) are locatated at the top (bottom) of the figure. Also shown is the trajectory of an electron as it diffuses from through the cavitons from the magnetosheath to the magnetosphere.

smaller scales do result. If $E^2 \approx 10^{-3}$, as suggested by *Treumann et al.* [1992], then $L_\perp \approx 0.96$ km and $L_\parallel \approx$ 4 km.

The collapse of the waves is eventually limited by the interaction of the waves with the distribution function, so that, depending on the ambient parameters, either the electrons or ions get heated. In the strong turbulence scenario discussed above the heating of particles is due to stochastic scattering by clumps of LH waves. The mechanism is similar to transit time damping. To estimate the relative importance of electron and ion damping, we assume that ion (electron) heating occurs when $\omega \approx \omega_{LH} \approx \alpha k_\perp V_i$ ($\omega \approx \alpha k_\parallel V_e$), where α is a numerical factor that emphasizes that damping may become important long before the waves can interact with the bulk of the distribution [*Sotnikov et al.*, 1978]. Note that since the ions are unmagnetized, they can damp the waves by interacting with either the parallel or perpendicular wave component. Since $L_\parallel \gg L_\perp$, the perpendicular component is the relevant one.

If the ratio of damping rates of protons to electrons is defined as

$$S = \frac{k_\perp V_i}{k_\parallel V_e} \quad (13)$$

then, using (6) and (10), we find

$$S = \sqrt{\frac{T_i}{T_e}} \frac{1}{k_\perp R} . \quad (14)$$

When $T_i \gg T_e$, as it the case at the magnetopause, ion heating is expected to dominate, since $k_\perp R \sim 1$. One point about the $T_i \gg T_e$ regime discussed here is that it is unclear whether energetic electron tails will be produced, as is the case when $T_e \geq T_i$ [*Papadopoulos*, 1992; *Shapiro et al.*, 1993a]. Numerical simulations can shed further light on this process [*Bingham et al.*, 1993].

4. Electron Transport Due to Modulational Instability

In the previous section we have concentrated on what could be termed "local" aspects of the MI such as particle heating and wave intensity levels. However, the MI also has important implications on larger scales. The collapse of the LH waves will result in a very patchy structure of the turbulence, with the waves confined to regions that are elongated substantially along the magnetic field. Figure 1 shows a sketch of the LH turbulence after MI. It is interesting to note that *Tsurutani et al.* [1989] show some evidence of this patchy structure in their observations of bursts of LH turbulence. Also, *Larson and Parks* [1992] have pointed out that ISEE 1 and 2 observations indicate that very fine scale structure exists at the magnetopause. They show evidence for large changes in electron fluxes (with energies of a few keV) on timescales of a second or less. In some cases a sequence of maxima in the electron flux was observed over a time of a few seconds. Such patchy structure would be expected from the MI.

While we showed in the previous section that electron heating due to modulationally unstable lower hybrid waves was small, the structure of the turbulence has significant implications for cross-field electron transport. We can calculate the cross-field electron diffusion coefficient by assuming that the result of the MI is a set of randomly spaced and phased soliton-like structures with dimensions $L_\perp$ and $L_\parallel$, as shown in Figure 1. If the transit time, $\tau = L_\parallel/V_e$, of an electron is much shorter than ω_{LH}^{-1} ($\tau \ll 1/\omega_{LH}$), an electron crossing such a structure will be displaced transversely by

$$\Delta x_\perp = \frac{cE}{B}\tau \leq L_\perp \ . \qquad (15)$$

Figure 1 shows the trajectory of an electron that interacts with a series of such soliton-like structures.

For the values discussed in the previous section, $\Delta x_\perp \approx 40$ km $\approx L_\perp$. The cross-field diffusion coefficient is

$$D = \frac{< \Delta x_\perp \Delta x_\perp >}{\tau_0} \ , \qquad (16)$$

where τ_0 is the time between encounters. As a result (in MKS units)

$$D = \frac{E^2}{B^2}\left(\frac{L_\parallel}{V_e}\right)\left(\frac{L_\parallel}{L_0}\right) \ \mathrm{m^2/s} \qquad (17)$$

where L_0 is the average soliton distance (i.e., $L_\parallel/L_0$ is the one-dimensional packing ratio). For the parameters discussed previously ($E^2 = 5\times10^{-5}\mathrm{V^2/m^2}$, $B = 15$ nT, $L_\parallel \approx 80$ km, $V_e \approx 5 \times 10^6$m/s), we find that

$$D = 9 \times 10^9\left(\frac{L_\parallel}{L_0}\right)\mathrm{m^2/s} \qquad (18)$$

which for $L_\parallel/L_0 \approx O(1)$ exceeds the rate required by *Sonnerup* [1980]. For the values discussed by *Treumann et al.* [1992], $E^2 = 10^{-3}$, but D is virtually unchanged since $L_\parallel$ is now much smaller.

If the quantity $(E^2/8\pi)(L_\parallel/L_0)$ is defined to be the average density of LH wave electric field needed for MI (equation (6)), then, on use of (11) is is possible to write (17) in the following more traditional form:

$$D \simeq \frac{T}{\sqrt{m_i m_e}}\sqrt{\frac{T_i}{T_e}}\frac{1}{\Omega_e} \sim D_B\sqrt{\frac{m_e}{m_i}\frac{T_i}{T_e}} \ \mathrm{m^2/s} \qquad (19)$$

where $D_B = T_i/eB_0$ is the Bohm diffusion coefficient (in MKS units) when $T_i \gg T_e$, as is the case at the magnetopause. However, we note that (17) containing the turbulent electric field energy and caviton scale length $L_\parallel$ as free parameters is in many ways more convenient for a comparison with observations.

Random scattering of ions by the LH cavitons also results in an effective collision frequency. In the ion frame of reference, energy goes from the electron drift sustaining the tangential discontinuity, to the LH waves and then is subsequently absorbed by the protons. The effective collision frequency due to this mechanism can be of order ω_{LH}, similar to quasi-linear theory [*Sotnikov et al.*, 1980; *Winske et al.*, 1991). This topic will be addressed in more detail elsewhere.

A final point should be made. Quasi-linear theory requires that the linear modes are preserved, and that they interact coherently with particles over length scales of many wavelengths. In the case under consideration, the linear modes of the homogeneous plasma have been destroyed by the wave collapse and the wave-particle interactions are no longer coherent but are due to cumulative transit effects. A statistical "stochastic" theory is then required to describe the resultant transport [*Morales and Lee*, 1975]. A formal theory, such as developed by *Krommes* [1984], results in transport that is quadratic in the electric field (i.e., $\propto E^2$), similar to (17). This similarity to quasi-linear theory is due to the fact that in both cases we consider the lowest-order terms. However, while quasi-linear theory requires that phase resonance be maintained over distances of many wavelengths, the stochastic theory is nonresonant.

5. Discussion and Conclusions

We have investigated the role of the modulational instability of lower hybrid waves at the magnetopause. For typical values of magnetopause lower hybrid turbulence, the modulational instability should be triggered, leading to ion heating and cross-field transport of electrons. The turbulence is predicted to have a very clumpy structure, so that the electron transport is due to their interaction with a series of finite sized regions of turbulence (or scattering centers). The cross-field diffusion coefficient exceeds that required by *Sonnerup* [1980].

One possible limitation of the present analysis is that the MI appears to be limited to the magnetosheath side of the magnetopause, whereas lower hybrid turbulence is seen to penetrate significant distances into the magnetosphere. In addition, from the perspective of diffusion, one would like the diffusing electrons to also penetrate into the magnetosphere. Unfortunately, the group velocity of the lower hybrid waves in the cross-field direction is rather small (of order the ion thermal speed), so that this issue needs to be addressed.

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